

Determination Method of Buckling Load for Eccentric Cementing Casing Based on Adjacent Balance Criterion

Guijie Yu¹, Ruihe Wang², Weiguo Li³

China University of Petroleum (East China), Qingdao, Shandong, China

¹yuguojie@upc.edu.cn; ²wangrh@upc.edu.cn; ³liwg@upc.edu.cn

Abstract- Eccentric cementing status always occurs in the period of well completion in complex formation. Under this condition, buckling failure of casing has greater impact on normal production. According to adjacent balance criterion and higher differential equation, a determination method of buckling load for eccentric cementing casing was established. This avoids complex modeling and discretization processes when determining the critical buckling load of casing, and geometrical and physical parameters can directly be put in the formula of critical buckling load. There exists 8.3% relative error, comparing critical load from the formula with the result by using finite element method. That means the results are accurate. This method has the advantage of direct calculation and is easy for engineering applications without complex modeling and discretization processes.

Keywords- Eccentric Cementing; Casing; Adjacent Balance Criterion; Higher Differential Equation; Buckling; Critical Load; FEM

I. INTRODUCTION

Eccentric cementing means there appears eccentric annulus between the casing and borehole wall because the casing in the borehole is not centered. When cementing, the mud cannot be completely replaced by cement slurry. Part of cement sheath between wall and casing after the solidification of cement contacts with casing closely, and other part contacts with underground liquid. The state calls eccentric cementing status (Fig. 1), which often occurs in horizontal well section^[1]. It is the condition that does not meet the requirements of cementing. This condition often appears in well completion of complex formation^[2]. Obviously, eccentric cementing status lies in axis of casing. The casing and cement contact with formation closely. On the ring of casing, only partially external circular surfaces of casing and cement contact with formation. Partially external circular surfaces of casing contacts with cement and formation, other meets with no borehole, annular space is full of formation liquid and mud. The load on casing is the hydrostatic pressure, which is the load of partially external circular surfaces by underground liquid (Fig. 2). The entire cement circle on the outer surface of casing could improve the collapsing strength^[3]. The paper^{[4]~[8]} studied the collapsing strength in common condition. In this paper, according to adjacent balance criterion, casing deformation of eccentric cementing status was studied and the analysis method for collapsing strength of eccentric cementing casing was confirmed by means of mathematical analysis solution^[9]. And the example was shown.

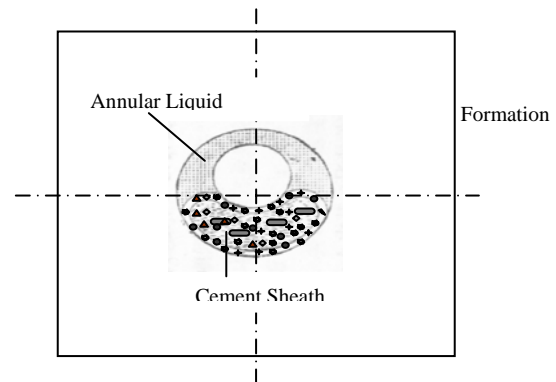


Fig. 1 Eccentric cementing chart

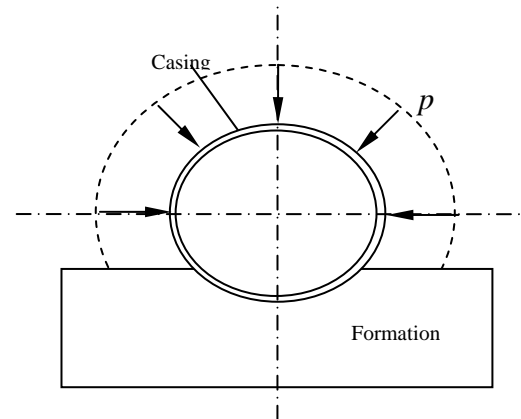


Fig. 2 Eccentric cementing mechanical model

II. SIMPLIFICATION OF MECHANICAL MODEL FOR ECCENTRIC CEMENTING STATUS CASING

Suppose there was considerable length for eccentric casing along axis (length of bare casing was beyond the range of constraint by cement), and it can be supposed as indefinite length. In order to simplify the issue, the physical dimensions of casing without cement were assumed: length as l along axis (considerable length), circumference size was expressed by centre angle of casing middle surface. That is $2\theta_0$. Because the size of bare casing along axis was big enough, there was no influence between constraint along axis and circle. When studying the buckling deformation of eccentric cementing status casing, take unit length of casing (ring) for study. The ring could be replaced by the circle in which average diameter of casing lay. Mechanical model was showed as Fig. 3.

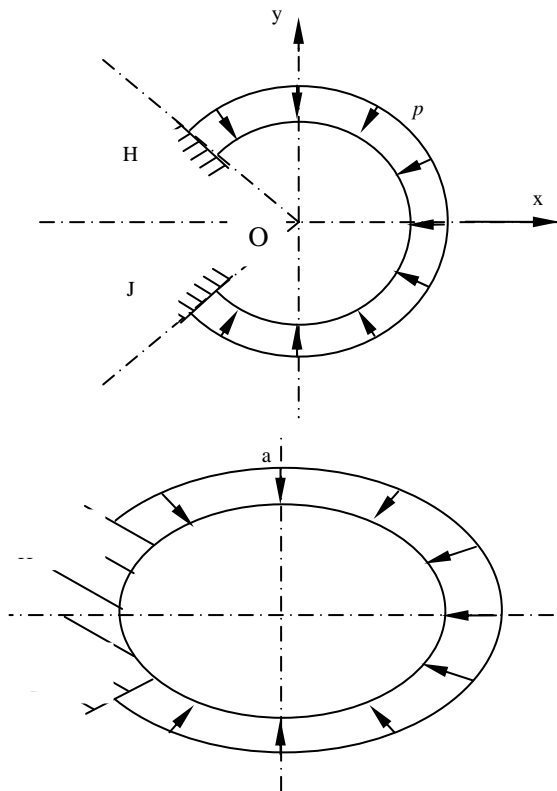


Fig. 3 Chart of mechanical model

III. RELATION BETWEEN THE DEFORMATION OF RING AND THE LOAD

As shown in Fig. 3b: unit length of the ring, average radius (the radius of middle surface) a , wall thickness t .

Since $\frac{t}{a} \ll 1$, the casing was thin-walled. Dots (Fig. 4) were expressed with polar coordinates (r, θ) .

Without deformation at ring, the geometrical relationship was:

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = r - a \\ (ds)^2 = (rd\theta)^2 \end{cases} \quad (1)$$

According to energy principle, under the load of hydrostatic pressure P , deformation appeared at the ring, and the equation between deformations of local fixed ring and load^[10] was:

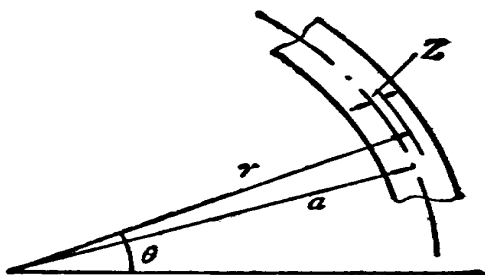


Fig. 4 Thin-walled dots

$$\begin{cases} \left[\frac{V'+W}{a} + \frac{1}{2} \left(\frac{V-W'}{a} \right)^2 \right]' + \frac{1}{Ea^2} \left(\frac{V-W'}{a} \right)' \\ - \left[\frac{V'+W}{a} + \frac{1}{2} \left(\frac{V-W'}{a} \right)^2 \right] \left(\frac{V-W'}{a} \right)' \\ - \frac{pa}{EA_0} \left(\frac{V-W'}{a} \right) = 0 \\ \frac{I}{A_0 a^2} \left(\frac{V-W'}{a} \right)' - \left[\frac{V'+W}{a} + \frac{1}{2} \left(\frac{V-W'}{a} \right)^2 \right] \\ - a \left\{ \left[\frac{V'+W}{a} + \frac{1}{2} \left(\frac{V-W'}{a} \right)^2 \right] \left(\frac{V-W'}{a} \right)' \right\}' \\ - \frac{pa}{EA_0} \left(\frac{V'+W}{a} \right) = \frac{pa}{EA_0} \end{cases} \quad (2)$$

In which:

V, W —Displacement component of middle surface of the ring along circumferential and radial, m;

E —Modulus of elasticity of the ring, Pa;

A_0 —Sectional area of the ring, m²;

I —Moment of inertia of section relative to middle surface, m⁴.

IV. BUCKLING ANALYSIS OF THE RING

Under the axisymmetric load, for the entire load P , there were two types of shape for the stability of ring. One was circular stability balance; the other was noncircular stability balance. Obviously, from circular stability balance to noncircular stability balance, there must be a balance in the form of critical state, which the load was corresponding to was called critical load, written as p_{cr} .

Assume circumferential and radial displacement component of circular stability balance as (V_0, W_0) , and circumferential and radial displacement component of noncircular stability balance as $(V_0 + \Delta v, W_0 + \Delta w)$. Circular configuration and noncircular configuration both met Equation (2). In which $(\Delta v, \Delta w)$ was infinite small incremental. For configuration of circular stability balance, there was:

$$V_0 = V_0' = W_0' = 0 \quad (3)$$

Then, take (V, W) to configuration of noncircular stability balance, (V, W) could be showed as following.

$$\begin{cases} V = \Delta v \\ W = W_0 + \Delta w \end{cases} \quad (4)$$

Take Equation (3) and (4) to Equation (2), and $(\Delta v, \Delta w)$ was considered. Omit high order, we got:

$$\begin{cases} EA_0 a^2 [(\Delta v)' + (\Delta w)'] \\ + EI [(\Delta v) - (\Delta w)']'' \\ - EA_0 a W_0 [(\Delta v) - (\Delta w)'] \\ - pa^3 [(\Delta v) - (\Delta w)'] = 0 \\ EI [(\Delta v) - (\Delta w)']''' \\ - EA_0 a^2 [(\Delta v)' + (\Delta w)']'' \\ - EA_0 a W_0 [(\Delta v) - (\Delta w)'] \\ - pa^3 [(\Delta v) + (\Delta w)] = 0 \end{cases} \quad (5)$$

Due to symmetry:

$$W_0 = -\frac{pa^2}{EA_0} \quad (6)$$

Take Equation (6) to Equation (5), the stability equation of local fixed ring impacted by hydrostatic pressure was obtained.

$$\begin{cases} A_0 a^2 [(\Delta v)' + (\Delta w)'] + \\ I [(\Delta v) - (\Delta w)']'' = 0 \\ EA_0 a^2 [(\Delta v)' + (\Delta w)'] \\ - EI [(\Delta v) - (\Delta w)']''' \\ + pa^3 [(\Delta v) + (\Delta w)] = 0 \end{cases} \quad (7)$$

According to adjacent balance criterion^[10], $(\Delta v, \Delta w)$ in Equation (7) was circumferential and radial displacement increment of middle surface, which was from circular stability to noncircular stability of local fixed ring. Clearly, $(\Delta v, \Delta w)$ was the function of centre angle θ of ring. According to the feature of structure symmetry and load symmetry, and $(\Delta v, \Delta w)$ was the function of centre angle θ , the function was the general solution for Equation (7), and it must meet the following boundary conditions.

$$\begin{cases} \Delta v|_{-\theta_0} = (\Delta v)'|_{-\theta_0} = 0 \\ \Delta w|_{-\theta_0} = (\Delta w)'|_{-\theta_0} = 0 \\ (\Delta w)'|_{\theta=0} = 0 \end{cases} \quad (8)$$

Suppose : $a_1 = EA_0 a^2, a_2 = EI, a_3 = pa^3$

$$\begin{cases} X = (\Delta v)' + (\Delta w) \\ Y = (\Delta v) - (\Delta w)' \end{cases} \quad (9)$$

Equation (7) can be rewritten as:

$$\begin{cases} a_1 X' + a_2 Y'' = 0 \\ a_1 X - a_2 Y''' + a_3 [X - Y'] = 0 \end{cases} \quad (10)$$

To Equation (7) and (10), analytic solutions can be obtained with high differential equation method.

Assume:

$$Z = \begin{pmatrix} X \\ Y \\ Y' \\ Y'' \end{pmatrix} \quad (11)$$

Then:

$$Z' = \begin{pmatrix} X' \\ Y' \\ Y'' \\ Y''' \end{pmatrix} \quad (12)$$

Equation (10) can be rewritten as first-order equations.

$$\begin{pmatrix} X' \\ Y' \\ Y'' \\ Y''' \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \frac{a_2}{a_1} \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -\frac{a_1 + a_3}{a_2} & 0 & \frac{a_3}{a_2} & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Y' \\ Y'' \end{pmatrix} = 0 \quad (13)$$

That was

$$Z' + AZ = 0 \quad (14)$$

In which A was

$$-A = \begin{pmatrix} 0 & 0 & 0 & -\frac{a_2}{a_1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{a_1 + a_3}{a_2} & 0 & -\frac{a_3}{a_2} & 0 \end{pmatrix} \quad (15)$$

Characteristic polynomial for Equation (14)

$$|\lambda I + A| = \lambda^2 (\lambda^2 + \frac{a_3}{a_2} + \frac{a_3}{a_1} + 1) = 0 \quad (16)$$

Characteristic value for Equation (16):

$$\begin{cases} \lambda_1 = 0 (\text{double root}) \\ \lambda_2 = \sqrt{\frac{a_3}{a_2} + \frac{a_3}{a_1} + 1} \cdot i \\ \lambda_3 = -\sqrt{\frac{a_3}{a_2} + \frac{a_3}{a_1} + 1} \cdot i = -\lambda_2 \end{cases} \quad (17)$$

So that three equations were acquired as following.

$$\begin{cases} (-A - \lambda_1 I)^2 u = 0 \\ (-A - \lambda_2 I) u = 0 \\ (-A - \lambda_3 I) u = 0 \end{cases} \quad (18)$$

Take $n_1 = 2, n_2 = 1, n_3 = 1$, and the basic solution for Equation (18) was:

$$\xi_1 = \alpha \begin{pmatrix} 1 \\ 0 \\ a_1 + a_3 \\ a_3 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \xi_2 = \gamma \begin{pmatrix} \frac{a_2}{a_1} \lambda_2 \\ 1 \\ \lambda_2 \\ \lambda_2^2 \end{pmatrix}, \xi_3 = \tau \begin{pmatrix} \frac{a_2}{a_1} \lambda_3 \\ 1 \\ \lambda_3 \\ \lambda_3^2 \end{pmatrix} \quad (19)$$

Suppose $\eta = (\eta_1, \eta_2, \eta_3, \eta_4)^T = \xi_1 + \xi_2 + \xi_3$, the relationship among $\alpha, \beta, \gamma, \tau$ was

$$\begin{cases} \alpha = \eta_1 + \frac{a_2 a_3 \eta_3 - a_2 (a_1 + a_3) \eta_1}{a_1 a_2 + a_2 a_3 + a_3 a_1} \\ \beta = \eta_2 - \frac{\eta_4}{\lambda_2^2} \\ \gamma = \frac{\eta_4 (a_1 a_2 + a_2 a_3 + a_3 a_1) + a_1 a_3 \eta_3 \lambda_2 - a_1 (a_1 + a_3) \eta_1 \lambda_2}{2 \lambda_2^2 (a_1 a_2 + a_2 a_3 + a_3 a_1)} \\ \tau = \frac{\eta_4 (a_1 a_2 + a_2 a_3 + a_3 a_1) - a_1 a_3 \eta_3 \lambda_2 + a_1 (a_1 + a_3) \eta_1 \lambda_2}{2 \lambda_2^2 (a_1 a_2 + a_2 a_3 + a_3 a_1)} \end{cases}$$

Take $\alpha, \beta, \gamma, \tau$ to Equation (19), the expression of ξ_1, ξ_2, ξ_3 can be obtained.

The form of solution for Equation (14) was

$$\phi(t) = \sum_{j=1}^3 e^{\lambda_j t} \left[\sum_{i=0}^{n_j-1} \frac{(-A - \lambda_j I)^i}{i!} \right] \xi_j = [I - tA] \xi_1 + e^{\lambda_2 t} \xi_2 + e^{\lambda_3 t} \xi_3 \quad (20)$$

Assume $\eta = (1, 0, 0, 0)^T$. Take $(0, 1, 0, 0)^T$, $(0, 0, 1, 0)^T$ and $(0, 0, 0, 1)^T$ to $\phi(t)$, and four linearly independent solutions $\phi_i(t)$ were got. Then from Equation (11)

$$Z = (X, Y, Y', Y'')^T = (\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)) C \quad (21)$$

In which $C = (c_1, c_2, c_3, c_4)^T$, constant vector. So

$$\begin{cases} X = a_1 a_3 c_1 + [a_2 a_3 - a_2 a_3 \cos \mu t] c_3 + \frac{\mu a_2 \sin \mu t}{a_1} c_4 \\ Y = a_1 (a_1 + a_3) t c_1 + c_2 + \left[a_2 (a_1 + a_3) t + \frac{a_1 a_3}{\mu} \sin \mu t \right] c_3 + (1 + \cos \mu t) c_4 \end{cases} \quad (22)$$

In which

$$\mu = \sqrt{\frac{a_3}{a_2} + \frac{a_3}{a_1} + 1}$$

Equation (9) can be rewritten as

$$\begin{pmatrix} \Delta v' \\ \Delta w' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \Delta v \\ \Delta w \end{pmatrix} + \begin{pmatrix} X \\ -Y \end{pmatrix} \quad (23)$$

Equation (23) was nonhomogeneous linear system of differential equations. The proper vector corresponding to the proper value ($\bar{\lambda}_1 = i$, $\bar{\lambda}_2 = -i$) of coefficient matrix

$\bar{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ met the following equation.

$$(\bar{\lambda} I - \bar{A}) \bar{\xi} = 0 \quad (24)$$

That is:

$$\begin{pmatrix} \bar{\lambda} & 1 \\ -1 & \bar{\lambda} \end{pmatrix} \begin{pmatrix} \bar{\xi}_1 \\ \bar{\xi}_2 \end{pmatrix} = 0 \quad (25)$$

$$\begin{cases} \bar{\lambda} \bar{\xi}_1 + \bar{\xi}_2 = 0 \\ -\bar{\xi}_1 + \bar{\lambda} \bar{\xi}_2 = 0 \end{cases} \quad (26)$$

Solution vector corresponding to it was

$$\begin{cases} \bar{\xi}_1 = i \\ \bar{\xi}_2 = 1 \end{cases} \quad (27)$$

and

$$\begin{cases} \bar{\xi}_1 = -i \\ \bar{\xi}_2 = 1 \end{cases} \quad (28)$$

So $\tau_1 = (i, 1)^T$, $\tau_2 = (-i, 1)^T$, and basic solutions for Equation (23) were

$$\bar{\Phi}(\theta) = \begin{pmatrix} i e^{i\theta} & -i e^{-i\theta} \\ e^{i\theta} & e^{-i\theta} \end{pmatrix} \quad (29)$$

Inverse of Equation (29) was

$$\bar{\Phi}^{-1}(\theta) = \begin{pmatrix} -\frac{i}{2} e^{-i\theta} & \frac{1}{2} e^{-i\theta} \\ \frac{i}{2} e^{i\theta} & \frac{1}{2} e^{i\theta} \end{pmatrix} \quad (30)$$

The form of general solution for nonhomogeneous linear system of differential equations (Equation (23)) was

$$\begin{pmatrix} \Delta v \\ \Delta w \end{pmatrix} = \bar{\Phi}(\theta) \bar{\Phi}^{-1}(-\theta_0) \eta + \bar{\Phi}(\theta) \int_{-\theta_0}^{\theta} \bar{\Phi}^{-1}(s) \begin{pmatrix} X(s) \\ -Y(s) \end{pmatrix} ds \quad (31)$$

What can be obtained according to the first and second term of Equation (8) was

$$\begin{pmatrix} \Delta v \\ \Delta w \end{pmatrix} = \int_{-\theta_0}^{\theta} \begin{pmatrix} \cos(\theta-s) & -\sin(\theta-s) \\ \sin(\theta-s) & \cos(\theta-s) \end{pmatrix} \begin{pmatrix} X(s) \\ -Y(s) \end{pmatrix} ds \quad (32)$$

$$0 = \begin{pmatrix} \Delta v \\ \Delta w \end{pmatrix} \bigg|_{\theta=-\theta_0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X(-\theta_0) \\ -Y(-\theta_0) \end{pmatrix} \quad (33)$$

So

$$\begin{cases} 0 = a_1 a_3 c_1 + (a_2 a_3 - a_2 a_3 \cos \mu \theta_0) c_3 - \frac{a_2 \mu}{a_1} \sin \mu \theta_0 c_4 \\ 0 = a_1 (a_1 + a_3) \theta_0 c_1 - c_2 + \left[a_2 (a_1 + a_3) \theta_0 + \frac{a_1 a_3}{\mu} \sin \mu \theta_0 \right] c_3 - (1 + \cos \mu \theta_0) c_4 \end{cases} \quad (34)$$

From Equation (32)

$$\Delta w = \int_{-\theta_0}^{\theta} [X(s) \sin(\theta-s) - Y(s) \cos(\theta-s)] ds$$

The expression for function $X(s)$ and $Y(s)$ can be seen in Equation (22).

What can be obtained according to the first and second term of Equation (8) was

$$\begin{aligned} 0 = \Delta v|_{\theta_0} &= [-a_1^2 \sin \theta_0 + a_1 (a_1 + a_3) \theta_0 \cos \theta_0] c_1 + (1 - \cos \theta_0) c_2 \\ &+ [-a_2 \sin \theta_0 + a_2 (a_1 + a_3) \theta_0 \cos \theta_0 + (\frac{a_1 a_3}{\mu} - a_2 a_3) \frac{\sin(\mu+1)\theta_0}{2(\mu+1)} - (a_2 a_3 + \frac{a_1 a_3}{\mu}) \frac{\sin(\mu-1)\theta_0}{2(\mu-1)}] c_3 \\ &+ [(1 - \cos \theta_0) + (\frac{1}{2} \frac{\mu_2}{2a_1} \frac{1 - \cos(\mu+1)\theta_0}{(\mu+1)} - (\frac{1}{2} + \frac{\mu_2}{2a_1}) \frac{1 - \cos(\mu-1)\theta_0}{(\mu-1)})] c_4 \end{aligned} \quad (35)$$

Third-order equations consist of four constant coefficients c_1, c_2, c_3, c_4 :

$$\begin{cases} 0 = a_1 a_3 c_1 + (a_2 a_3 - a_2 a_3 \cos \mu \theta_0) c_3 - \frac{a_2 \mu}{a_1} \sin \mu \theta_0 c_4 \\ 0 = a_1 (a_1 + a_3) \theta_0 c_1 - c_2 + \left[a_2 (a_1 + a_3) \theta_0 + \frac{a_1 a_3}{\mu} \sin \mu \theta_0 \right] c_3 - (1 + \cos \mu \theta_0) c_4 \\ 0 = [-a_1^2 \sin \theta_0 + a_1 (a_1 + a_3) \theta_0 \cos \theta_0] c_1 + (1 - \cos \theta_0) c_2 \\ + [-a_2 \sin \theta_0 + a_2 (a_1 + a_3) \theta_0 \cos \theta_0 + (\frac{a_1 a_3}{\mu} - a_2 a_3) \frac{\sin(\mu+1)\theta_0}{2(\mu+1)} - (a_2 a_3 + \frac{a_1 a_3}{\mu}) \frac{\sin(\mu-1)\theta_0}{2(\mu-1)}] c_3 \\ + [(1 - \cos \theta_0) + (\frac{1}{2} \frac{\mu_2}{2a_1} \frac{1 - \cos(\mu+1)\theta_0}{(\mu+1)} - (\frac{1}{2} + \frac{\mu_2}{2a_1}) \frac{1 - \cos(\mu-1)\theta_0}{(\mu-1)})] c_4 \end{cases} \quad (36)$$

Or be written as

$$\begin{aligned} A_{11} c_1 + A_{12} c_2 + A_{13} c_3 + A_{14} c_4 &= 0 \\ A_{21} c_1 + A_{22} c_2 + A_{23} c_3 + A_{24} c_4 &= 0 \\ A_{31} c_1 + A_{32} c_2 + A_{33} c_3 + A_{34} c_4 &= 0 \end{aligned} \quad (37)$$

In which

$$\begin{cases} A_{11} = a_1 a_3, A_{12} = 0, A_{13} = a_2 a_3 - a_2 a_3 \cos \mu \theta_0, A_{14} = -\frac{a_2 \mu}{a_1} \sin \mu \theta_0 \\ A_{21} = a_1 (a_1 + a_3) \theta_0, A_{22} = -1, A_{23} = a_2 (a_1 + a_3) \theta_0 + \frac{a_1 a_3}{\mu} \sin \mu \theta_0, A_{24} = -(1 + \cos \mu \theta_0) \\ A_{31} = -a_1^2 \sin \theta_0 + a_1 (a_1 + a_3) \theta_0 \cos \theta_0, A_{32} = 1 - \cos \theta_0, \\ A_{33} = -a_2 \sin \theta_0 + a_2 (a_1 + a_3) \theta_0 \cos \theta_0 + (\frac{a_1 a_3}{\mu} - a_2 a_3) \frac{\sin(\mu+1)\theta_0}{2(\mu+1)} - (a_2 a_3 + \frac{a_1 a_3}{\mu}) \frac{\sin(\mu-1)\theta_0}{2(\mu-1)} \\ A_{34} = (1 - \cos \theta_0) + (\frac{1}{2} \frac{\mu_2}{2a_1} \frac{1 - \cos(\mu+1)\theta_0}{(\mu+1)} - (\frac{1}{2} + \frac{\mu_2}{2a_1}) \frac{1 - \cos(\mu-1)\theta_0}{(\mu-1)}) \end{cases}$$

What was obtained was the proportional relationship among the four coefficients. Firstly, make $c_4 = 1$, so c_1 and c_3 were acquired from the first and third equation. Then take them to

the second equation, we got c_2 . The expression of $\begin{pmatrix} \Delta v \\ \Delta w \end{pmatrix}$

was acquired by taking them to Equation (32).

On the basis of analysis, Δv and Δw were the function of E, A_0, a, I, P and θ , it meant they were the function of a_1, a_2, a_3 and θ in analyzing. Under the axial symmetry load, to make the ring be in critical state from circular stability balance to noncircular stability balance, there must have the following equations for all θ .

$$\begin{cases} \Delta v = 0 \\ \Delta w = 0 \end{cases} \quad (38)$$

Meanwhile, critical load p_{cr} of buckling failure of ring was gained.

V. EXAMPLES

Finite element model of eccentric cementing status was showed in Fig. 5.

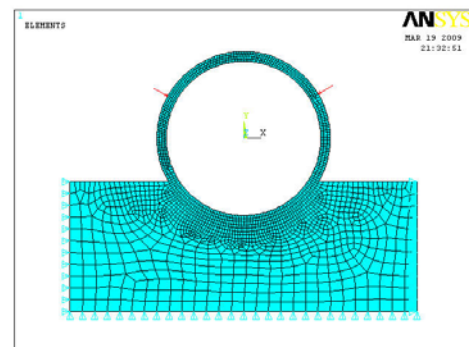


Fig. 5 Element analysis model of structure

Basic parameter of casing: grade of steel was N80; external diameter (D) was 139.7 and wall thickness was 7.72; modulus of elasticity was 216GPa; poisson's ratio was 0.25. Stratum parameter: modulus of elasticity was 216GPa;

poisson's ratio was 0.3. angle of bare pipe θ_0 equaled to 120° symmetrically. The load was external pressure P .

Take the basic parameters above to the coefficients of Equation (7) and (9), we got:

$$\begin{aligned} a &= 69.85 \times 10^{-3} (\text{m}); \quad A_0 = 7.72 \times 10^{-6} (\text{m}^2); \\ I &= 3.8342 \times 10^{-11} (\text{m}^4); \quad a_1 = EA_0 a^2 = 7909.8713 (\text{Nm}^2); \\ a_2 &= EI = 0.8052 (\text{Nm}^2); \quad a_3 = 0.06985 p (\text{N/m}^2). \end{aligned}$$

After taking a_1, a_2, a_3 and θ_0 to Equation (36), when given P an initial value p_0 , the first maximal value p_{cr} of P was 155.1 MPa through trial.

Under the condition above, the hydrostatic pressure p_{cr} was 155.1 MPa when theoretical eccentric cementing appeared stability disruption.

Finite element method:

According to the physical model established with the parameter above, take plane182 for discretization to the structure. Make horizontal displacement of left and right side limited, as well as vertical displacement of underside. Casing would be affected by external pressure p .

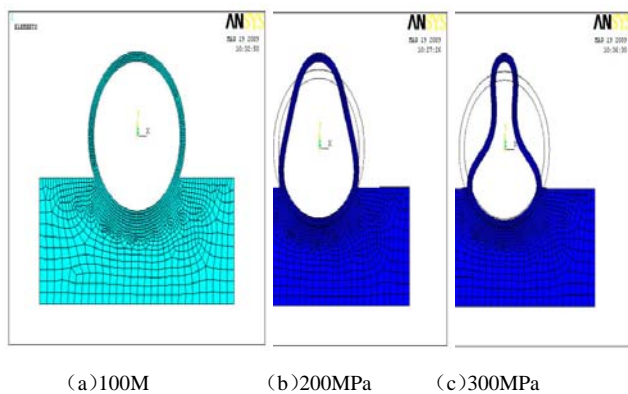


Fig. 6 Deflection of eccentric cementing casing

There existed no relative displacement between casing and layer. Finite element model could be seen in Fig. 5. There were 1780 elements and 2008 nodes in total.

In Fig. 6a to 6c, structural deforming graphs were showed under the condition of external pressure 100MPa, 200MPa and 300MPa. It can be seen from the graphs that both Fig. 6b and Fig. 6c were in unstable state. The relationship between radial displacement and external pressure on the peak of casing was given by Fig. 7, from which the break of displacement appeared when external pressure was 168MPa. That was to say, critical pressure was 168Mpa when it was unstable.

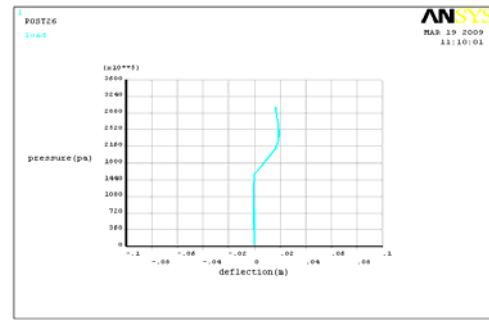


Fig. 7 Radial displacement vs. pressure of casing at acme

Compare finite element results with theoretical results, error of calculation was: $\frac{168 - 155.1}{155.1} = 8.3\%$. Theoretical results were close to finite element results. And the accuracy of theoretical analysis was validated.

VI. CONCLUSION

The method, buckling analysis of eccentric cementing status casing, is an easy and valid way for the study of collapse resistance. It is not as complex as the physical finite element modeling process, but the stable load can be acquired by directly entering the relevant parameters. For the convenience of calculation, accuracy and convenience for engineering applications, it is easy for theory to be applied to engineering project.

REFERENCES

- [1] Xu Huifeng. Drilling Technical Manual (3) (Cement) [M]. Petroleum industry Press, Beijing: 1990. 7.
- [2] Wu Jiang, Hao Junfang. Permit eccentricity of well cementation casing [J]. Petroleum Drilling Techniques, 1989, 17(4): 42-47.
- [3] Song Ming, Influencing Tendencies of Cement Sheath on Casing Retaining Capacity [J], Oil Drilling & Production Technology, 2002, 24(4):7-9.
- [4] Cui xiaobing, Zhang Hong, Li Zihou, An Wenhua, Song Zhi. Research on Reasonable Free Segment Length of Deep Oil Well Casing String [J], Acta Petrolei Sinica, 1997, 18(1): 91-95.
- [5] Zhang Zhiquan, Wu Jiazhong, Luo Yue. Analysis and Countermeasures of Casing Deformation Causes in Gangxi Oil field [J], Journal of Jiangnan Petroleum Institute, 2002, 24(4): 83-84.
- [6] Fang Jun. Analysis of Loading Propetty of Casing and Cement Sheath Under Nonuniform Geologic Stress [J], Journal of the University of Petroleum, 1995,19(6):52-57.
- [7] Gao Xueshi, Fang Jun. Calculation Method for Collapse of Oil Casing Restrained by Concrete [J]. Journal of the University of Petroleum, 1994, 18(3):39-46.
- [8] Ma Xu, Lab Test of Destruction of Cement Sheath Outside Casing, China Petroleum Machinery, 2000, 28(4):15-18.
- [9] Li Weiguo, Tong Dengke. Numerical Computing Methods [M]. University of Petroleum Press, Shandong: 262-291(2008.10).
- [10] Chen Tiejun, Shen Huizhong. The Bucking of Structure [M]. Shanghai Science and Technology Literature Press, Shanghai: 111-120(1993, 12).